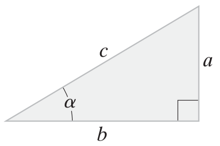
TRIGONOMETRÍA



Las funciones trigonométricas para un triángulo rectángulo son

$$\text{sen}\ \alpha = \frac{1}{\csc\alpha} = \frac{a}{c}, \qquad \cos\alpha = \frac{1}{\sec\alpha} = \frac{b}{c}, \qquad \tan\alpha = \frac{1}{\cot\alpha} = \frac{a}{b}.$$

El seno y el coseno satisfacen la relación

$$\text{sen}^2\ \alpha + \cos^2\alpha = 1,$$

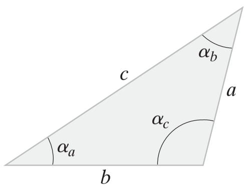
y el seno y el coseno de la suma y la resta de dos ángulos satisfacen

$$\text{sen}(\alpha+\beta) = \text{sen}\ \alpha\cos\beta + \cos\alpha\ \text{sen}\ \beta,$$

$$\text{sen}(\alpha-\beta) = \text{sen}\ \alpha\cos\beta - \cos\alpha\ \text{sen}\ \beta,$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \text{sen}\ \alpha\ \text{sen}\ \beta,$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \text{sen}\ \alpha\ \text{sen}\ \beta.$$



La **ley de los cosenos** para un triángulo arbitrario es

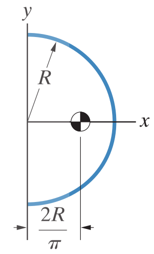
$$c^2 = a^2 + b^2 - 2ab\cos\alpha_c,$$

y la **ley de los senos** es

$$\frac{\text{sen}\ \alpha_a}{a} = \frac{\text{sen}\ \alpha_b}{b} = \frac{\text{sen}\ \alpha_c}{c}.$$

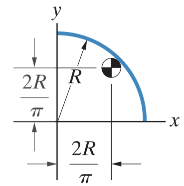
PROPIEDADES DE LÍNEAS

Longitud $= \pi R$



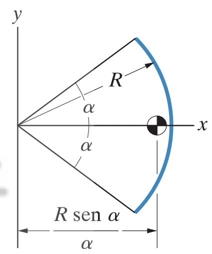
Arco semicircular

Longitud $= \frac{1}{2}\pi R$



Arco de un cuarto de círculo

Longitud $= 2\alpha R$



Arco circular

Vector cartesiano

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitud

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direcciones

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$
$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Producto punto

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$

Producto cruz

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Vector cartesiano de posición

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

Vector cartesiano de fuerza

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Momento de una fuerza

$$M_o = Fd$$
$$\mathbf{M}_o = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Momento de una fuerza alrededor de un eje específico

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplificación de un sistema de fuerza y par

$$\mathbf{F}_R = \Sigma\mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

Equilibrio

Partícula

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

Cuerpo rígido-dos dimensiones

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_O = 0$$

Cuerpo rígido-tres dimensiones

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$
$$\Sigma M_{x'} = 0,\ \Sigma M_{y'} = 0,\ \Sigma M_{z'} = 0$$

Fricción

Estática (máxima) $F_s = \mu_s N$

Cinética $F_k = \mu_k N$

Centro de gravedad

Partículas o partes discretas

$$\bar{r} = \frac{\Sigma\tilde{r}W}{\Sigma W}$$

Cuerpo

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

Momentos de inercia de área y masa

$$I = \int r^2\,dA \qquad I = \int r^2\,dm$$

Teorema de los ejes paralelos

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

Radio de giro

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

Trabajo virtual

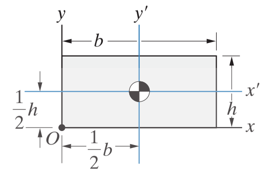
$$\delta U = 0$$

PROPIEDADES DE ÁREA

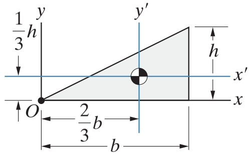
Área $= bh$

$$I_x = \frac{1}{3}bh^3, \qquad I_y = \frac{1}{3}hb^3, \qquad I_{xy} = \frac{1}{4}b^2h^2$$

$$I_{x'} = \frac{1}{12}bh^3, \qquad I_{y'} = \frac{1}{12}hb^3, \qquad I_{x'y'} = 0$$



Área rectangular

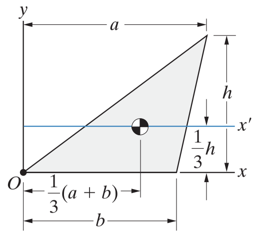


Área triangular

Área $= \frac{1}{2}bh$

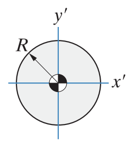
$$I_x = \frac{1}{12}bh^3, \qquad I_y = \frac{1}{4}hb^3, \qquad I_{xy} = \frac{1}{8}b^2h^2$$

$$I_{x'} = \frac{1}{36}bh^3, \qquad I_{y'} = \frac{1}{36}hb^3, \qquad I_{x'y'} = \frac{1}{72}b^2h^2$$



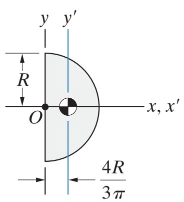
Área triangular

Área $= \frac{1}{2}bh \qquad I_x = \frac{1}{12}bh^3, \qquad I_{x'} = \frac{1}{36}bh^3$



Área circular

Área $= \pi R^2 \qquad I_{x'} = I_{y'} = \frac{1}{4}\pi R^4, \qquad I_{x'y'} = 0$



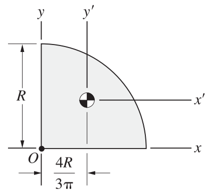
Área semicircular

Área $= \frac{1}{2}\pi R^2 \qquad I_x = I_y = \frac{1}{8}\pi R^4, \qquad I_{xy} = 0$

$$I_{x'} = \frac{1}{8}\pi R^4, \qquad I_{y'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4, \qquad I_{x'y'} = 0$$

Área $= \frac{1}{4}\pi R^2 \qquad I_x = I_y = \frac{1}{16}\pi R^4, \qquad I_{xy} = \frac{1}{8}R^4$

$$I_{x'} = I_{y'} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)R^4, \qquad I_{x'y'} = \left(\frac{1}{8} - \frac{4}{9\pi}\right)R^4$$

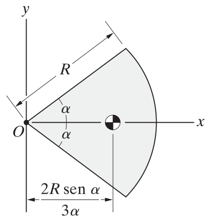


Área de un cuarto de círculo

Área $= \alpha R^2$

$$I_x = \frac{1}{4}R^4\left(\alpha - \frac{1}{2}\text{sen } 2\alpha\right), \qquad I_y = \frac{1}{4}R^4\left(\alpha + \frac{1}{2}\text{sen } 2\alpha\right),$$

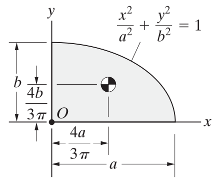
$$I_{xy} = 0$$



Sector circular

Área $= \frac{1}{4}\pi ab$

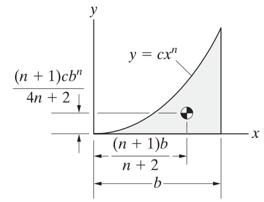
$$I_x = \frac{1}{16}\pi ab^3, \qquad I_y = \frac{1}{16}\pi a^3 b, \qquad I_{xy} = \frac{1}{8}a^2b^2$$



Área de un cuarto de elipse

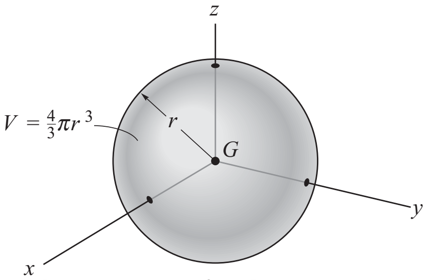
Área $= \frac{cb^{n+1}}{n+1}$

$$I_x = \frac{c^3b^{3n+1}}{9n+3}, \qquad I_y = \frac{cb^{n+3}}{n+3}, \qquad I_{xy} = \frac{c^2b^{2n+2}}{4n+4}$$



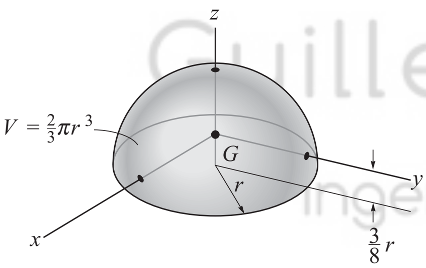
Enjunta (Sector general)

CENTROS DE VOLUMENES Y MOMENTOS DE INERCIA DE SÓLIDOS



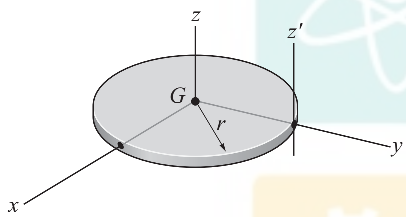
Esfera

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



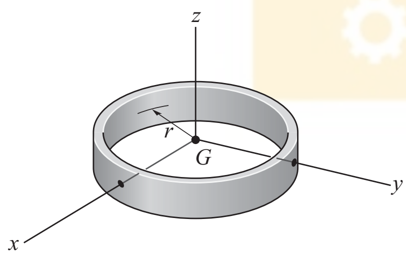
Semiesfera

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



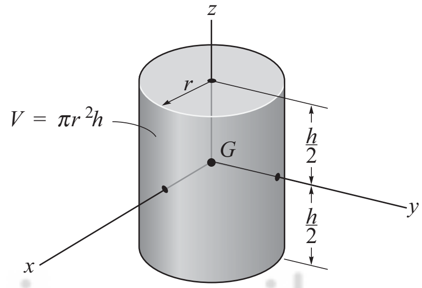
Disco circular delgado

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



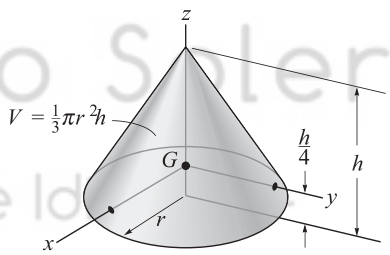
Anillo delgado

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



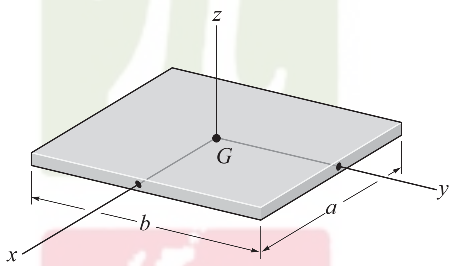
Cilindro

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



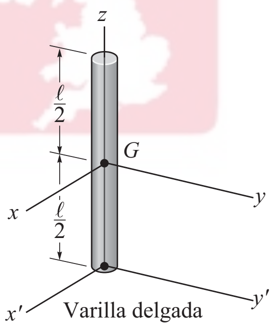
Cono

$I_{xx} = I_{yy} = \frac{3}{80} m\,(4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$



Placa delgada

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Varilla delgada

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$